

2023 MOCK 1 FURTHER / ELECTIVE MATHEMATICS 2

SECTION A

[48 marks]

Answer **all** questions in this section.

All questions carry **equal** marks.

eemmaths

1. A binary operation Δ is defined on the set R of real numbers by $m\Delta n = m + n + 10$. Find:
 - (a) the identity element.
 - (b) the inverse of 2 and -5 under Δ .
2. Solve the simultaneous equations;
 $\log(x - 1) + 2 \log y = 2 \log 3,$
 $\log x + \log y = \log 6.$
3. If α and β are the roots of the quadratic equation $4x^2 + 7x + 3 = 0$, find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.
4. The sum of the first n terms of a sequence is given by $S_n = \frac{5}{2}n^2 + \frac{5}{2}n$, find
 - (i) the n th term of the sequence.
 - (ii) write down the first four terms of the sequence.
5. A committee consists of 7 members from the AOGA group and 5 members from the MEWO group. If a chairman and a secretary of the committee are selected at random, what is the probability that they are of:
 - (a) the same group.
 - (b) different groups.

6. The table shows the relation between weekly advertisement (**P**) and the number of items sold (**Q**) by Mr. Obuobi in his shop.

P	0	7	3	5	6	1	8	9	4	2
Q	0	107	25	101	63	81	42	94	12	105

Calculate, correct to **four decimal places**, the Spearman's rank correlation coefficient.

7. (a) Show that the triangle whose vertices have position vectors $4\mathbf{i} + 2\mathbf{j}$, $5\mathbf{i} + 3\mathbf{j}$ and $2\mathbf{i} + 5\mathbf{j}$ is isosceles.
- (b) Given that $\mathbf{a} = k\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j}$, find the value of k for which $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.
8. Three forces whose magnitudes are 15N, 9N and 34N act along the vectors $-3\mathbf{i} + 4\mathbf{j}$, $13\mathbf{i}$ and $8\mathbf{i} - 15\mathbf{j}$ respectively. Find, correct to **one decimal place**,
- (a) the magnitude of the resultant force.
- (b) the direction of the resultant force.

SECTION B

[52 marks]

Answer **four** questions **only** from this section **with at least one** question from **each** part.

All questions carry **equal** marks.

PART I

PURE MATHEMATICS

9. (a) Express $\frac{3x^2+7x+28}{x^3+x^2+7x}$ in partial fractions.
- (b) If $7x^2 - 8y^2 + 2xy - 3x = 101$, find $\frac{dy}{dx}$ at the point $(-2, 5)$.
10. (a) (i) Write down the matrix **A** of the linear transformation
 $A: (x, y) \rightarrow (-2x - y, -5x + 3y)$
- (ii) If $B = \begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix}$, find;
- (α) $B^2 - A^2$
- (β) matrix **H**, if $H = B^2A$
- (b) Without using mathematical tables or calculator, evaluate

$$\frac{\frac{3}{2}\log 27 - 3\log 5\sqrt{5}}{\log 0.6}.$$
11. (a) A line passes through the centres of the circles $x^2 + y^2 + 6x + 3y + 4 = 0$ and $x^2 + y^2 + x - 2y - 3 = 0$.
- (i) Find the equation of the line.
- (ii) If the centres of the two circles in (a) are the ends of a diameter of another circle, find the equation of that circle.
- (b) If α and β are the roots of $3x^2 - 5x - 1 = 0$, find an equation whose roots are $\left(\alpha - \frac{1}{\beta}\right)$ and $\left(\beta - \frac{1}{\alpha}\right)$.

PART II
STATISTICS AND PROBABILITY

12. The table below shows the distribution of marks of some students in a test conducted by Sir Poborsky.

Marks	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
Number of Students	2	7	6	7	8	5	3	2

Using an assumed mean of 65.5,

- (a) calculate, correct to **two decimal places**, the mean of the distribution.
- (b) calculate, correct to **three decimal places**, the standard deviation.
13. (a) A box contains 7 blue and 4 red marbles. Three marbles are selected at random, one after the other without replacement from the box, find in **two decimal places**, the probability that;
- (i) they are of the same colour.
- (ii) two blue and one red marbles are selected.
- (b) In Aburi Girls' SHS, 3 out of every 8 students taught by Sir Abraham and Sir Bright, attend AOGA meeting. If 5 students are selected at random from the school, find, correct to **two decimal places**, the probability that:
- (i) **exactly two** attend AOGA meeting.
- (ii) **less than half** of them attend AOGA meeting.
- (iii) **at least** three of them attend AOGA meeting.

PART III

VECTORS AND MECHANICS

14. The position vectors of the points A, B, C, D relative to the origin, O , are given by $\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 4 \\ n \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} p \\ -1 \end{pmatrix}$, where n and p are constants. Find the:
- (a) unit vector in the direction of $\overrightarrow{AB}$
 - (b) value of n for which angle $AOC = 90^\circ$.
 - (c) values of p for which the length of $\overrightarrow{AD}$ is $\sqrt{37}$ units.
15. (a) A school bell weighing 75N is suspended by two ropes which make angles 42° and 67° with the horizontal, respectively. Find the tensions in the rope, correct to **two decimal places**.
- (b) A uniform plank AB , 10m long and of mass, mkg , rest on two supports A and B . A load of $8kg$ is placed on the plank at C , such that $|AC| = 3.5m$. The reaction at B is 100N. If the system remains in equilibrium under the action of these forces, calculate;
- (i) value of m ; in **one decimal place**.
 - (ii) the reaction at A .
- [Take $g = 10ms^{-2}$]

END OF PAPER